Kinematics & Dynamics of Linkages

Lecture 15: Mechanical Advantage

Power in a Systems

Mechanical System

Dot product of the force vector F and the velocity V

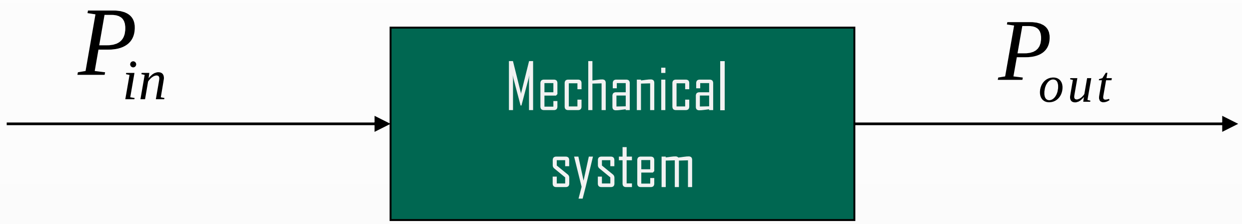
$$P = F.V = F_x V_x + F_y V_y$$

Rotating Systems

Dot product of the torque T and the angular velocity V

$$P = T\omega$$

Energy loss



$$losses = P_{in} - P_{out}$$

Mechanical efficiency $\varepsilon = \dfrac{P_{out}}{P_{in}}$

Ideal Case

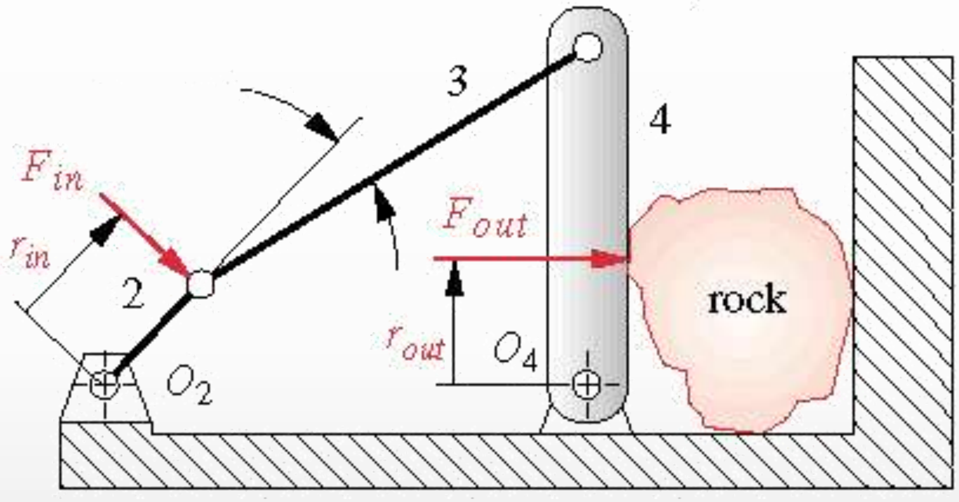
$$P_{in} = P_{out}$$

$$P_{in} = T_{in}\omega_{in}$$

$$P_{out} = T_{out}\omega_{out}$$

$$T_{in}\omega_{in} = T_{out}\omega_{out}$$

$$\frac{T_{out}}{T_{in}} = \frac{\omega_{in}}{\omega_{out}}$$

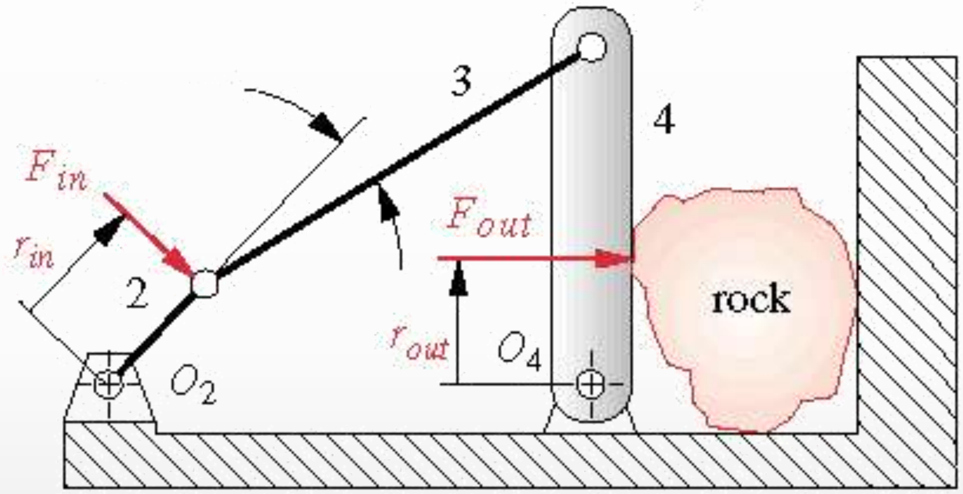


Ideal Case

Input Force F_{in}

Output Force F_{out}

$$F_{out} = \frac{T_{out}}{r_{out}} \qquad F_{in} = \frac{T_{in}}{r_{in}}$$



Mechanical Advantage

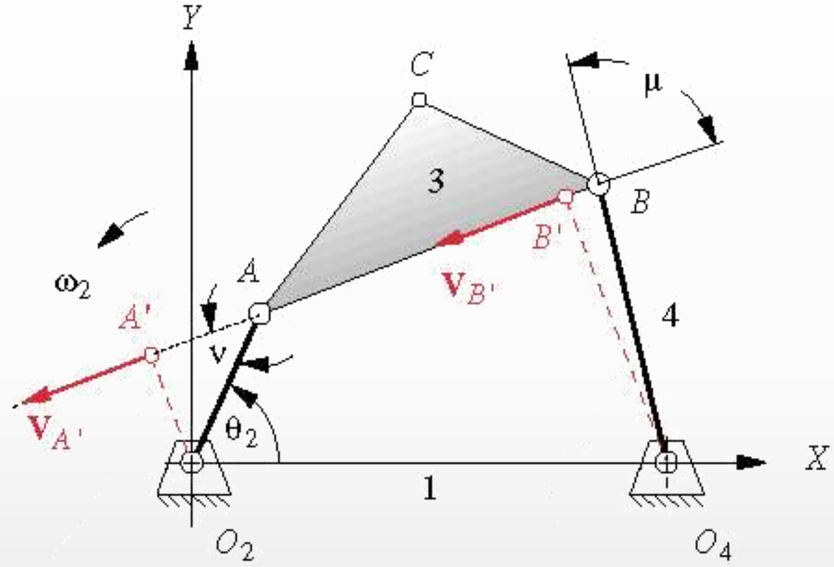
$$m_A = \frac{F_{out}}{F_{in}} = \frac{T_{out}}{T_{in}}\frac{r_{in}}{r_{out}} = \frac{\omega_{in}}{\omega_{out}}\frac{r_{in}}{r_{out}} = \frac{V_{in}}{V_{out}}$$

Mechanical Advantage – 4 Bar Linkage

From Velocity Analysis

$$\omega_4 = \frac{a\omega_2}{c}\frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)}$$

But $\dfrac{\omega_4}{\omega_2} = \dfrac{O_2 A \sin \nu}{O_4 B \sin \mu}$



Which gives

$$m_A = \frac{\omega_{in}}{\omega_{out}}\frac{r_{in}}{r_{out}} = \frac{V_{in}}{V_{out}} = \frac{c \sin \mu}{a \sin \nu}\frac{r_{in}}{r_{out}}$$

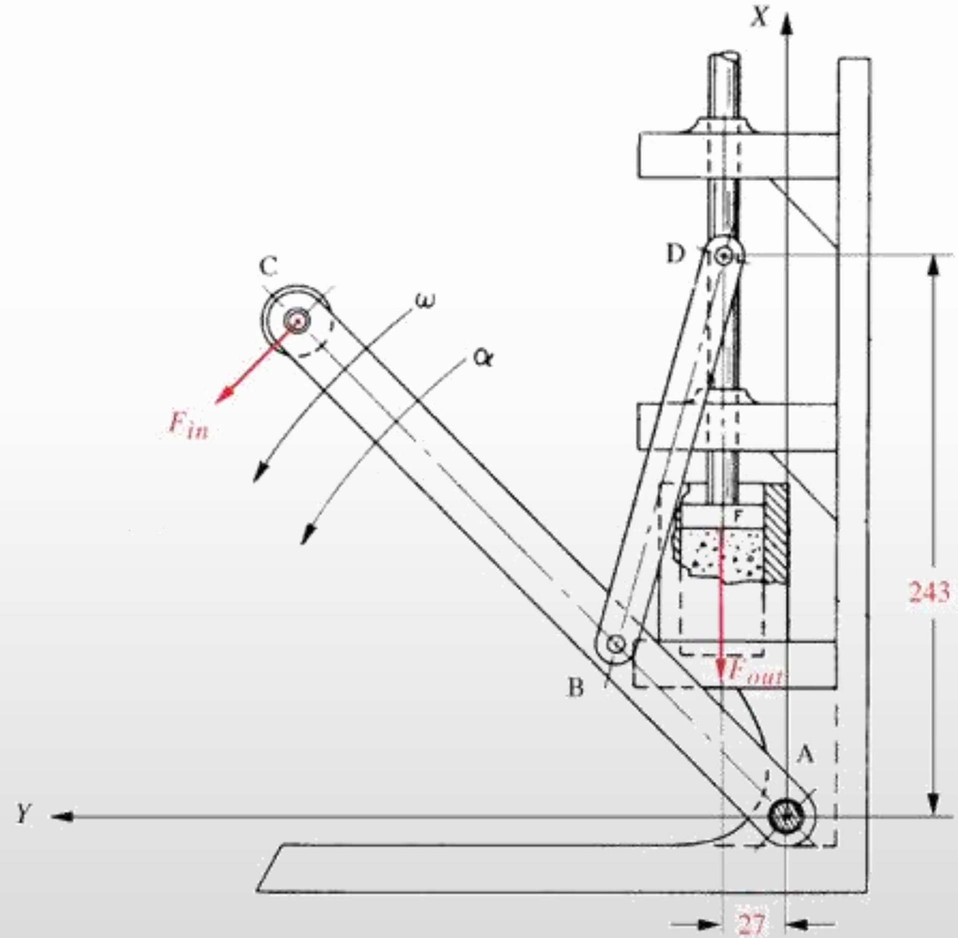
Example

Calculate the mechanical advantage for the powder compaction mechanism.

AB = 105 mm @ 44°

BD =172 mm

AC = 301 mm @ 44°



Example – Solution

From the drawing (offset Slider–Crank):

$\theta_2 = 44^o$

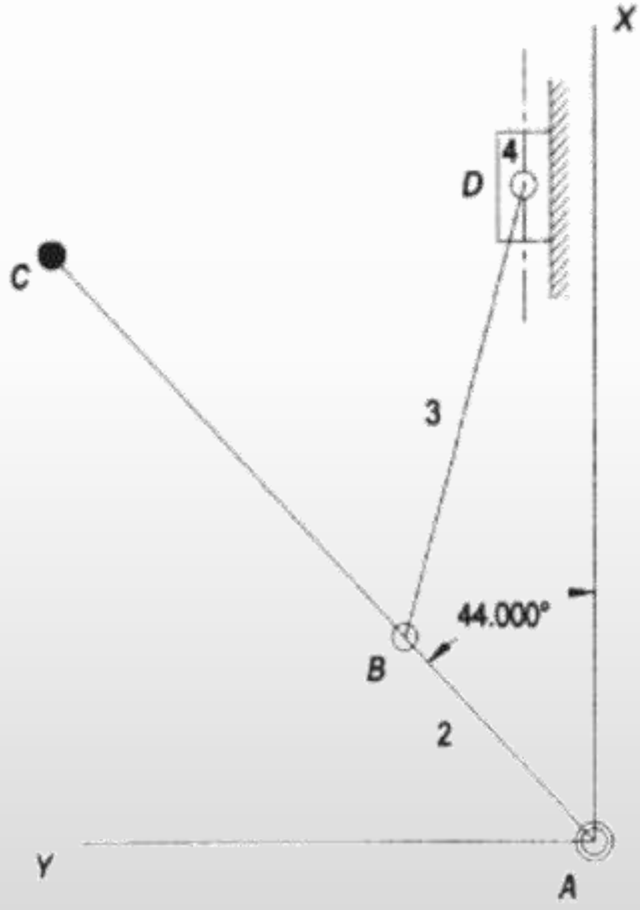
r_{in} = AC = 301 mm

a = AB = 105 mm

b = BD = 172 mm

c = 27 mm

d = 243 mm

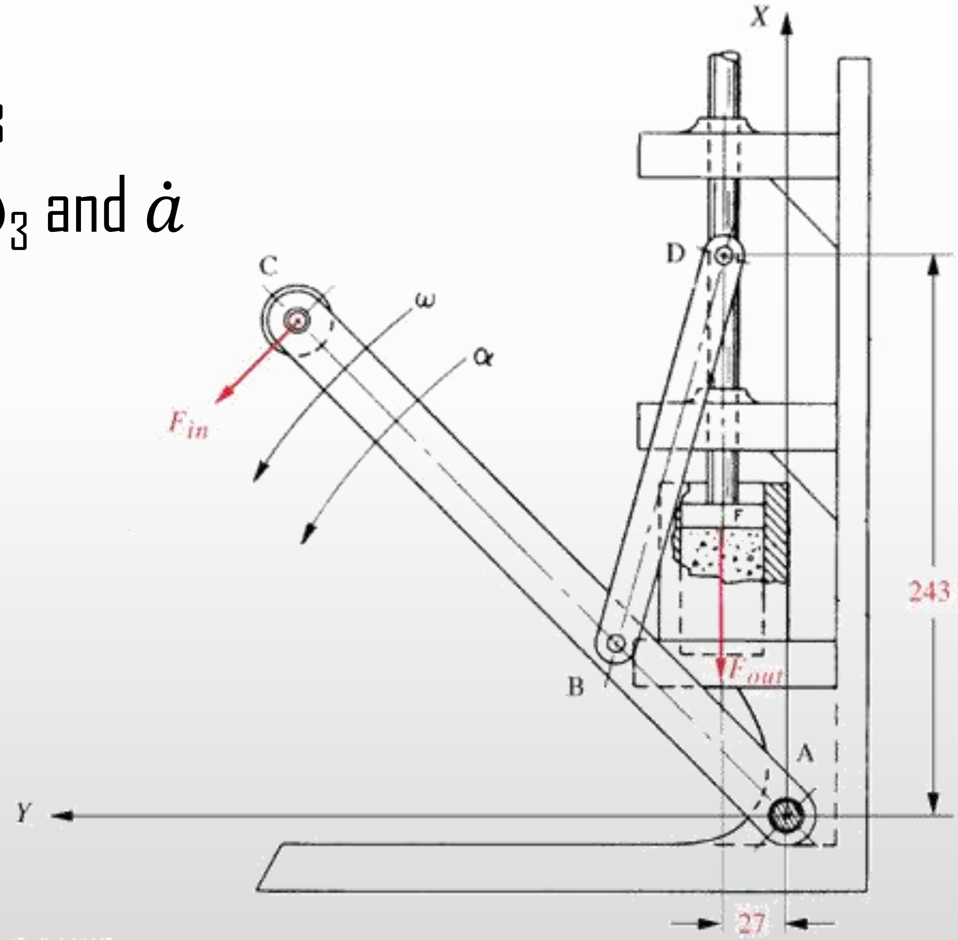


Example – Steps

Step 1: Position analysis > find θ_3

Step 2: Velocity analysis > find ω_3 and $\dot{\alpha}$

Step 3: Calculate $m_A = \frac{V_{in}}{V_{out}}$



Example – Step 1

Find θ_3 from previous position analysis

a = 105 mm, b = 172 mm,

c = 27 mm, d = 243 mm, θ_2= 44°

$$\theta_3 = \arcsin\left(-\frac{a\sin\theta_2 - c}{b}\right) + \pi = \arcsin\left(-\frac{105\sin 44^o - 27}{172}\right) + \pi = 164.5^o$$

Example – Step 2

Find ω_3 and $\dot{a}$ from previous velocity analysis

$$\omega_3 = \frac{a\cos\theta_2}{b\cos\theta_3}\,\omega_2 = -0.456\,\omega_2$$

$$\dot{d} = -a\omega_2\sin\theta_2 + b\omega_3\sin\theta_3 = -93.9\,\omega_2$$

Example – Step 3

Calculate the mechanical advantage

a = 105 mm, b = 172 mm, c = 27 mm, d = 243 mm, θ_2= 44o

$$V_{out} = \left|\dot{d}\right|$$

$$V_{in} = \left|\omega_{in} r_{in}\right| = \left|AC\ \omega_2\right|$$

$$m_A = \frac{V_{in}}{V_{out}} = \frac{301\omega_2}{93.9\omega_2} = 3.2$$